

Necessary Instantiation and Logical Independence in Wittgenstein's *Tractatus*

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Abstract

In this paper, I distinguish three interpretations of the logical independence of atomic facts in Wittgenstein's *Tractatus*, which I shall refer to as pairwise, one-step, and unrestricted combinatorial independence. I argue that one-step and unrestricted combinatorial independence are incompatible with the thesis that objects are necessarily instantiated and that the *Tractatus* is committed to this thesis on textual, ontological, and semantic grounds. It follows that pairwise independence is the most plausible interpretation of logical independence in the *Tractatus*.

Keywords: Wittgenstein; *Tractatus*; logical independence; necessary instantiation; ontology; possible worlds

1. Introduction

The doctrine of the logical independence of atomic facts occupies a central place in the ontology of Wittgenstein's *Tractatus*.¹ However, there is disagreement about how this doctrine should be understood.² In this paper, I distinguish three interpretations of logical independence: pairwise independence, one-step independence, and unrestricted combinatorial independence.

On the pairwise interpretation, the obtainment or non-obtainment of any atomic fact has no logical implications for the obtainment or non-obtainment of any distinct atomic fact. On the one-step interpretation, any atomic fact can obtain or fail to obtain while the obtainment or non-obtainment of every other atomic fact remains unchanged. On the unrestricted combinatorial interpretation, every combination of the obtainment and non-obtainment of atomic facts corresponds to a possible world.

I argue that the *Tractatus* is committed to the thesis that every object is necessarily instantiated on textual, ontological, and semantic grounds. I then develop a reductio showing that this thesis is incompatible with one-step independence. Since unrestricted combinatorial independence entails one-step independence, the reductio also applies to the unrestricted combinatorial interpretation of logical independence. This provides a new argument against both interpretations and thereby strengthens the case for the pairwise interpretation of logical independence.

The paper is structured as follows. Section 2 introduces the possible-worlds framework adopted in this paper, presents the three interpretations of logical independence, and outlines the textual basis for each. Section 3 clarifies the thesis of necessary instantiation of objects and develops the reductio showing that one-step independence is incompatible with the thesis of necessary instantiation.

¹ Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, trans. D. F. Pears and B. F. McGuinness (Routledge, 2001).

² For discussions of logical independence in the *Tractatus*, see Fabrice Correia, "Logical Dependence and Independence in the *Tractatus*," in *Wittgenstein and the Future of Philosophy: A Reassessment after 50 Years (Contributions of the 24th International Wittgenstein Symposium)*, Vol. 1, eds. Rudolf Haller and Klaus Puhl (Austrian Ludwig Wittgenstein Society, 2001), pp. 155–161; Ralf Funke, "Independence vs. Compossibility," in *Image and Imaging in Philosophy, Science, and the Arts (Contributions of the 33rd International Wittgenstein Symposium)*, eds. Elisabeth Nemeth, Richard Heinrich, and Wolfram Pichler (Austrian Ludwig Wittgenstein Society, 2010), pp. 95–97; and Raymond Bradley, *The Nature of All Being: A Study of Wittgenstein's Modal Atomism* (Oxford University Press, 1992), who famously critically discusses this thesis as "the myth of independence" (p. 101).

Section 4 argues that the *Tractatus* is committed to the thesis that every object is necessarily instantiated. Section 5 addresses possible objections. Section 6 concludes the paper.

2. Possible Worlds and Logical Independence

Throughout this paper, I employ a possible-worlds framework.³ Relative to any possible world, each atomic fact either obtains or fails to obtain. Moreover, every atomic fact obtains in at least one possible world. A possible world is identified with the collection of atomic facts that obtain in it. Different possible worlds therefore differ only with respect to which atomic facts obtain. The actual world is the possible world consisting of the atomic facts that in fact obtain. The ontological status of objects is central to the *Tractatus* and is set out in the following propositions:

2.021 Objects make up the substance of the world. That is why they cannot be composite.

2.024 Substance is what subsists independently of what is the case.

2.0271 Objects are what is unalterable and subsistent; their configuration is what is changing and unstable.

On the reconstruction adopted in this paper, the domain of objects is therefore common to all possible worlds. Modal change across possible worlds consists exclusively in variation in which atomic facts obtain, rather than in the domain of objects. The *Tractatus* explicitly endorses the doctrine that atomic facts are logically independent:

2.061 States of affairs are independent of one another.

Here, states of affairs are understood as atomic facts. Proposition 2.061 does not specify what the logical independence of atomic facts amounts to. Three interpretations may be distinguished. The first interpretation is pairwise independence. According to this interpretation, neither the obtainment nor the non-obtainment of any atomic fact logically determines the obtainment or non-obtainment of any distinct atomic fact. This interpretation is naturally supported by proposition 2.062:

2.062 From the existence or non-existence of one state of affairs it is impossible to infer the existence or nonexistence of another.

Further support for the pairwise interpretation is provided by Wittgenstein's discussion of elementary propositions, which represent atomic facts:

4.211 It is a sign of a proposition's being elementary that there can be no elementary proposition contradicting it.

5.134 One elementary proposition cannot be deduced from another.

Proposition 6.37 likewise states:

6.37 There is no compulsion making one thing happen because another has happened. The only necessity that exists is *logical* necessity.

³ For reconstructions of the ontology of the *Tractatus* in terms of possible worlds and logical space, see Ralf Funke, "Independence vs. Compossibility," in *Image and Imaging in Philosophy, Science, and the Arts* (Contributions of the 33rd International Wittgenstein Symposium), eds. Elisabeth Nemeth, Richard Heinrich, and Wolfram Pichler (Austrian Ludwig Wittgenstein Society, 2010), pp. 95–97; María Cerezo, "Possibility and Logical Space in the *Tractatus*," *International Journal of Philosophical Studies* 20, no. 5 (2012): 645–659; and Oliver Thomas Spinney, "Logical Form and Logical Space in Wittgenstein's *Tractatus*," *Synthese* 200, no. 1 (2022): 1–23.

Taken together, these propositions naturally support the pairwise interpretation of logical independence. The second interpretation may be called one-step independence. This interpretation is suggested most directly by proposition 1.21:

1.21 Each item can be the case or not the case while everything else remains the same.

Within the possible-worlds framework adopted in this paper, the one-step interpretation may be formulated as follows. Given any possible world, removing a single obtaining atomic fact while leaving every other obtaining atomic fact unchanged yields another possible world. Likewise, given any possible world, adding a single non-obtaining atomic fact while leaving every other obtaining atomic fact unchanged yields another possible world.

One-step independence entails pairwise independence. Suppose that pairwise independence fails. Two cases suffice. First, suppose that the obtaining of one atomic fact entails the obtaining of another. Then, given a possible world in which both atomic facts obtain, removing the latter while leaving every other atomic fact unchanged would not yield a possible world. Second, suppose that the obtaining of one atomic fact entails the non-obtaining of another. Then, given a possible world in which the former obtains and the latter does not, adding the latter while leaving every other obtaining atomic fact unchanged would not yield a possible world. The remaining two cases, in which the non-obtainment of one atomic fact entails either the obtainment or the non-obtainment of another, are analogous. In each case, one-step independence fails. Thus, if pairwise independence fails, one-step independence also fails. It follows that one-step independence entails pairwise independence.

The converse, however, does not hold. The absence of entailment between the obtainment or non-obtainment of distinct atomic facts does not guarantee one-step independence. To see this, consider three atomic facts, F , G , and H , and suppose that the only possible worlds are $\{F\}$, $\{G\}$, $\{H\}$, and $\{F, G, H\}$. Pairwise independence holds, since for every pair of atomic facts, all four combinations of their obtainment and non-obtainment occur in some possible world. Nevertheless, one-step independence fails. Starting from the possible world $\{F\}$, adding G while leaving everything else unchanged yields $\{F, G\}$, which is not a possible world. Thus, pairwise independence does not entail one-step independence.

The third interpretation is the unrestricted combinatorial interpretation, according to which every combination of obtaining and non-obtaining atomic facts corresponds to a possible world. This interpretation is naturally suggested by proposition 4.27:

4.27 For n states of affairs, there are $K_n = [2^n]$ possibilities of existence and non-existence. Of these states of affairs any combination can exist and the remainder not exist.⁴

The unrestricted combinatorial interpretation entails one-step independence. For if every combination of obtaining and non-obtaining atomic facts corresponds to a possible world, then, given any possible world, every combination that differs from it with respect to only a single atomic fact also corresponds to a possible world. The converse entailment, however, holds only under additional assumptions. For example, if the set of atomic facts is finite, then from any possible world each of the 2^n combinations of obtaining and non-obtaining atomic facts can be reached in finitely many one-step changes. Hence each of the 2^n combinations corresponds to a possible world. In the general case, by contrast, one-step independence guarantees only closure under finite variations and is therefore weaker than unrestricted combinatorialism.

The three interpretations therefore form a hierarchy of increasing logical strength: unrestricted

⁴ For ease of exposition, I replace Wittgenstein's original summation notation, which expresses the number of possibilities as the sum of the binomial coefficients from 0 to n , by its mathematically equivalent expression 2^n .

combinatorial independence entails one-step independence, which in turn entails pairwise independence. None of the converse implications holds in general. As the foregoing discussion has shown, each interpretation finds textual support in the *Tractatus*. The textual support considered in this section therefore does not by itself determine which interpretation provides the most plausible account of the doctrine of logical independence.

In the next section, I clarify the thesis of necessary instantiation and develop a reductio showing that this thesis is incompatible with one-step independence. Since unrestricted combinatorial independence entails one-step independence, the reductio applies *a fortiori* to the unrestricted combinatorial interpretation.⁵ Thus, if the *Tractatus* is committed to the thesis of necessary instantiation, the reductio provides a principled basis for regarding the pairwise interpretation as the most plausible account of logical independence.⁶

3. Necessary Instantiation and the Reductio

According to the *Tractatus*, atomic facts are combinations of objects:

2.01 A state of affairs (a state of things) is a combination of objects (things).

2.03 In a state of affairs objects fit into one another like the links of a chain.

Accordingly, an object is a constituent of an atomic fact just in case it is part of the combination of objects constituting that atomic fact. Let us say that an object is instantiated in a possible world if and only if it is a constituent of at least one obtaining atomic fact in that world. An object is necessarily instantiated if and only if it is instantiated in every possible world. Consider the thesis that every object is necessarily instantiated. I shall refer to this thesis as the thesis of necessary instantiation. The purpose of the present section is not to argue that the *Tractatus* is committed to this thesis, but rather to show that it is incompatible with the one-step interpretation of logical independence. Whether the *Tractatus* is in fact committed to necessary instantiation is the subject of the next section.

The following reductio shows that the one-step interpretation of logical independence is incompatible with the thesis of necessary instantiation. Suppose the thesis of necessary instantiation holds. Consider an arbitrary object *a* and a possible world *w*. Since *a* is necessarily instantiated, there is at least one obtaining atomic fact in *w* that instantiates *a*. Let $S = \{F_1, \dots, F_n\}$ be the set of obtaining atomic facts in *w* that instantiate *a*.⁷

According to the one-step interpretation of logical independence, given any possible world and any obtaining atomic fact in that world, there exists a possible world in which that atomic fact does not obtain while every other atomic fact remains unchanged. Beginning with *w*, first apply this principle to F_1 . The resulting world is itself a possible world, so the principle applies equally to it. Repeating this

⁵ The incompatibility of unrestricted combinatorial independence with necessary instantiation also follows independently of the reductio. For if every combination of obtaining and non-obtaining atomic facts corresponds to a possible world, then, for any object, there is a combination in which none of the atomic facts instantiating that object obtains. In the corresponding possible world, the object is therefore not instantiated.

⁶ For the claim that the *Tractatus* endorses an interpretation corresponding to pairwise independence, see Peter M. Simons, "Logical and Ontological Independence in the *Tractatus*," in *Ethics: Foundations, Problems and Applications*, eds. Edgar Morscher and Rudolf Stranzinger (Hölder-Pichler-Tempsky, 1981), pp. 464–467. For a contrasting argument that the *Tractatus* endorses an interpretation corresponding to unrestricted combinatorial independence, see Fabrice Correia, "Logical Dependence and Independence in the *Tractatus*," in *Wittgenstein and the Future of Philosophy: A Reassessment after 50 Years* (Contributions of the 24th International Wittgenstein Symposium), Vol. 1, eds. Rudolf Haller and Klaus Puhl (Austrian Ludwig Wittgenstein Society, 2001), pp. 155–161.

⁷ The assumption that, in any possible world, an object is instantiated by only finitely many atomic facts is discussed in Section 5.

process for each member of S , we obtain the following sequence of possible worlds. After the first application, we obtain a possible world w_2 in which F_1 does not obtain, while $F_2, \dots, F_n$ do obtain. Since w_2 differs from w only in the non-obtainment of F_1 , the only obtaining atomic facts in w_2 that instantiate a are $F_2, \dots, F_n$. After the second application, we obtain a possible world w_3 , in which neither F_1 nor F_2 obtains, while $F_3, \dots, F_n$ do obtain. Accordingly, the only obtaining atomic facts in w_3 that instantiate a are $F_3, \dots, F_n$. Proceeding in this manner, after the k -th application we obtain a possible world w_{k+1} in which the first k members of S fail to obtain, while the remaining members do obtain and are the only obtaining atomic facts in w_{k+1} that instantiate a . After the n -th application, we obtain a possible world w_{n+1} in which no member of S obtains. It follows that no atomic fact in w_{n+1} instantiates a . Hence, a is not instantiated in w_{n+1} . This contradicts the thesis of necessary instantiation.

The reductio therefore establishes that one-step independence and the thesis of necessary instantiation cannot both be maintained. As noted above, since unrestricted combinatorial independence entails one-step independence, the reductio applies *a fortiori* to the unrestricted combinatorial interpretation. If, as I shall argue in the next section, the *Tractatus* is committed to the thesis of necessary instantiation, then neither one-step independence nor the stronger unrestricted combinatorial interpretation can provide the correct account of the doctrine of logical independence.

4. In Defense of the Thesis of Necessary Instantiation

The reductio developed in the preceding section depends upon ascribing the thesis of necessary instantiation to the *Tractatus*. In this section, I defend that attribution on the basis of textual, ontological, and semantic considerations.

Textual Considerations

Proposition 2.0131 provides strong textual support for the thesis that every object must occur in at least one obtaining atomic fact. Wittgenstein writes:

2.0131 A spatial object must be situated in infinite space. (A spatial point is an argument-place.) A speck in the visual field, though it need not be red, must have some colour: it is, so to speak, surrounded by colour-space. Notes must have some pitch, objects of the sense of touch some degree of hardness, and so on.

These examples concern obtainment rather than mere possibility. A spatial object must occupy some location in space. A visual object need not be red, but it must actually possess some determinate colour. Likewise, a tone must have some determinate pitch, and a tactile impression must possess some determinate degree of hardness. Each example presents an object that, although it need not possess one particular property, must nevertheless possess some determinate property of the relevant kind. Moreover, Wittgenstein concludes the series of examples with "and so on," indicating that they are intended as illustrations of a more general principle rather than as an exhaustive list. Taken together, these examples therefore suggest a general principle: every object must occur in at least one obtaining atomic fact. Further textual support is provided by proposition 2.0122.

Wittgenstein writes:

2.0122 Things are independent in so far as they can occur in all possible situations, but this form of independence is a form of connexion with states of affairs, a form of dependence. (It is impossible for words to appear in two different roles: by themselves, and in propositions.)

The structure of the first remark is significant. Its first clause concerns possible states of affairs. The second, introduced by the adversative "but," immediately contrasts this independence with a form of dependence through connection with states of affairs. Since the second clause is introduced as a contrast, it is unlikely that Wittgenstein intends merely to repeat the preceding claim about possible

situations. Rather, it is natural to understand the dependence in question as concerning obtaining rather than merely possible states of affairs. This reading is reinforced by the following remark. Wittgenstein compares objects with words. Words cannot occur both within propositions and independently of propositions. Since words clearly occur within propositions, they cannot occur independently of them. The analogy suggests that the same holds for objects: they must occur in obtaining atomic facts. Taken together, these passages provide textual support for attributing the doctrine of necessary instantiation to the *Tractatus*.

Ontological Considerations

The textual evidence is reinforced by the broader ontological framework of the *Tractatus*.

Wittgenstein writes:

1 The world is all that is the case.

1.1 The world is the totality of facts, not of things.

Moreover, proposition 2.021 states that objects form the substance of the world. If objects form the substance of the world, then they belong to the world, for it would be difficult to make sense of the substance of the world not belonging to the world. Since the world is the totality of facts, objects belong to that totality. But how can objects belong to the totality of facts? The most natural answer is that they belong to it by virtue of occurring in at least one obtaining atomic fact. The following remarks by Wittgenstein point in the same direction:

2.022 It is obvious that an imagined world, however different it may be from the real one, must have something—a form—in common with it.

2.023 Objects are just what constitute this unalterable form.

All possible worlds share the same form, and this form consists of the objects. Thus, the objects belong to every possible world. Each possible world is understood as the totality of the facts obtaining in that world. The foregoing question therefore arises for every possible world: how do objects belong to its totality of obtaining facts? The same answer applies: objects belong to the totality of obtaining facts only by occurring in at least one obtaining atomic fact.

This answer is further motivated by Wittgenstein's account of states of affairs. According to proposition 2.01, states of affairs are combinations of objects. Hence, objects contribute to the constitution of the world, understood as the totality of facts, only through their occurrence in obtaining atomic facts. An object that occurs in no such fact would make no contribution whatsoever to the totality of facts that constitutes the world. It would therefore be difficult to understand in what sense such an object could nevertheless form part of the substance of the world. Moreover, the thesis that objects are necessarily instantiated is supported by Wittgenstein's account of the logical form of objects. Wittgenstein writes:

2.0141 The possibility of its occurring in states of affairs is the form of an object.

He also writes:

2.0123 If I know an object I also know all its possible occurrences in states of affairs. (Every one of these possibilities must be part of the nature of the object.) A new possibility cannot be discovered later.

These remarks show that the logical form of an object is constituted by the totality of its possible occurrences in states of affairs. If none of those possibilities were realized in any obtaining atomic fact, the object's logical form would find no realization in the facts constituting the world. Yet the

Tractatus identifies an object's logical form with precisely those possibilities of occurrence and introduces objects as the substance of the world. It is therefore difficult to see how an object whose logical form finds no realization in the facts constituting the world could nevertheless belong to the substance of the world. The doctrine of the logical form of objects thus provides further support for ascribing the thesis of necessary instantiation to the *Tractatus*.

Semantic Considerations

The thesis of necessary instantiation is also supported by Wittgenstein's account of names and elementary propositions. Wittgenstein writes:

3.203 A name means an object. The object is its meaning [...].

4.21 The simplest kind of proposition, an elementary proposition, asserts the existence of a state of affairs.

4.22 An elementary proposition consists of names. It is a nexus, a concatenation, of names.

Since elementary propositions represent possible states of affairs by combining names of objects, the meaningful occurrence of a name in an elementary proposition depends upon the logical form of the object it represents. The logical form of an object determines the possible atomic facts in which it occurs and thereby the elementary propositions containing its name that represent those possible atomic facts. These are precisely the meaningful elementary propositions containing that name.

Now suppose that there exists an object *a* that occurs in no obtaining atomic fact. Then the world, understood in accordance with proposition 1 as all that is the case, does not manifest the logical form of *a*. Since the logical form of *a* consists in the possibilities of its occurrence in atomic facts, the world's failure to manifest that form implies that it does not ground the distinction between elementary propositions containing the name of *a* that represent possible atomic facts and those that represent no possible atomic fact at all. The world therefore does not ground the distinction between an elementary proposition containing *a* that is false because it represents a non-obtaining possible atomic fact and one that is meaningless because it represents no possible atomic fact at all.

This difficulty disappears if every object occurs in at least one obtaining atomic fact. The occurrence of an object in an obtaining atomic fact manifests its logical form, thereby determining the range of elementary propositions in which its name can occur meaningfully. The semantics of the *Tractatus* thus provides further support for attributing the doctrine of necessary instantiation to Wittgenstein.

The textual, ontological, and semantic considerations presented in this section provide a cumulative case for attributing the doctrine of necessary instantiation to the *Tractatus*.

5. Objections

The preceding discussion gives rise to several possible objections. In this section, I consider three. The first two concern the reductio developed in Section 3, while the third concerns the attribution of the thesis of necessary instantiation to the *Tractatus* argued for in Section 4.

Objection 1: The iterative application of proposition 1.21 is unwarranted

One might argue that proposition 1.21 licenses only a single application. It does not follow that the proposition may be applied successively so as to eliminate all obtaining atomic facts instantiating a given object.

Response

This objection is unpersuasive. Proposition 1.21 is not formulated as a principle governing only the actual world, but as a general principle concerning logical possibility. Each application of proposition 1.21 yields another possible world. Since that world is itself a possible world, proposition 1.21 applies

to it no less than to the original world. The successive application of proposition 1.21 therefore requires no additional principle beyond proposition 1.21 itself. If the proposition may be applied once, it may equally be applied again in the possible world thereby obtained.

Objection 2: The reductio presupposes that every object is instantiated only finitely many times

The reductio proceeds by successively applying proposition 1.21 to every obtaining atomic fact instantiating a given object. This construction presupposes that, for any object, there are only finitely many obtaining atomic facts in which it occurs. However, the *Tractatus* nowhere explicitly states that an object can occur in only finitely many obtaining atomic facts. If an object were instantiated in infinitely many obtaining atomic facts, the successive elimination envisaged by the reductio could never be completed. The argument would therefore appear to depend upon a finiteness assumption that is not established by the *Tractatus*.

Response

This objection correctly identifies an assumption that is not explicitly stated in the *Tractatus*. The text nowhere asserts that every object occurs in only finitely many obtaining atomic facts. The reductio therefore depends upon a finiteness assumption that cannot be derived from the text alone.

Although the *Tractatus* nowhere explicitly endorses finitism, it displays a consistent methodological preference for finite symbolic procedures over completed infinite totalities. This preference is reflected, first, in Wittgenstein's account of operations, where he writes:

5.2523 The concept of successive applications of an operation is equivalent to the concept 'and so on'.

Wittgenstein thus conceives of operations as rules for indefinite successive application rather than as giving rise to completed infinite totalities. At no point is the completed totality of all applications required. The rule licenses indefinite continuation without presupposing a completed infinite totality.

The significance of proposition 5.2523 extends beyond Wittgenstein's conception of operations. The expression "and so on" might naturally be understood as referring to a completed infinite totality. Wittgenstein, however, identifies the concept of "and so on" with that of the successive application of an operation, thereby excluding such an understanding. The concept expressed by "and so on" therefore adds no content over and above the rule governing successive application. Although Wittgenstein does not formulate the point in these terms, this is naturally understood in light of the distinction between potential and actual infinity: the expression "and so on" is analyzed in terms of a rule permitting indefinite continuation rather than as referring to a completed infinite totality.

The same methodological preference for finitely specified rules rather than completed infinite totalities is also reflected in Wittgenstein's account of numbers:

6.021 A number is the exponent of an operation.

Numbers are introduced as exponents of operations. Their characterization therefore depends upon the successive application of an operation rather than upon a completed infinite totality. Each number corresponds to a particular iteration of an operation. The sequence of numbers is generated through repeated application of a rule. Accordingly, the sequence is not presented as a completed infinite totality. Rather, what is fundamental is the finitely specified rule of successive generation.

In both cases, Wittgenstein formulates his account in terms of the unrestricted application of a finitely specified rule rather than a completed infinite totality. Taken together, these examples suggest a finitistic orientation in the *Tractatus*. The supposition that an object occurs in infinitely many obtaining atomic facts is in tension with this orientation.

Moreover, many commentators have interpreted Wittgenstein's later philosophy of mathematics as rejecting the conception of completed infinite totalities in favor of indefinitely repeatable finite

procedures.⁸ Although Wittgenstein's later philosophy of mathematics cannot be read back into the *Tractatus*, it supports the interpretation that the finitistic tendency already present in the *Tractatus* is deliberate rather than accidental.

Indeed, if Wittgenstein rejects the conception of completed infinite totalities in mathematics, it would be surprising if his ontology in the *Tractatus* allowed for a completed infinite totality of obtaining atomic facts instantiating a single object. For such a totality could naturally be regarded as numbered, thereby yielding a completed infinite collection of numbers and introducing precisely the kind of completed infinite totality that Wittgenstein rejects in his later reflections on the foundations of mathematics. The reductio's finitistic assumption is therefore not ad hoc, but fits naturally within Wittgenstein's broader philosophical outlook.

Nevertheless, it does not seem to fit with proposition 4.2211:

4.2211 Even if the world is infinitely complex, so that every fact consists of infinitely many states of affairs and every state of affairs is composed of infinitely many objects, there would still have to be objects and states of affairs.

Yet infinite complexity is introduced only as a hypothetical assumption. The proposition therefore does not straightforwardly commit Wittgenstein to the logical possibility of such complexity. Whether the hypothesis is intended as a genuine modal possibility or merely as a concessive assumption is itself an interpretative question.

Moreover, even if proposition 4.2211 is taken to introduce a genuine modal possibility, it is more subtle than it may initially appear. Wittgenstein does not merely consider infinitely many states of affairs, but immediately couples this with every state of affairs being composed of infinitely many objects. The latter is most naturally read as requiring an infinite domain of objects, since states of affairs are combinations of objects. On that reading, since the domain of objects is infinite, it does not follow that at least one object is instantiated by infinitely many obtaining atomic facts. Thus, even understood as a genuine modal possibility, proposition 4.2211 does not contradict the reductio's finitistic assumption. Whether Wittgenstein's coupling is intended to prevent the inference that there is an object instantiated by infinitely many obtaining atomic facts is a further interpretative question.

If there is a possible world in which an object is instantiated by infinitely many obtaining atomic facts, then the reductio's finitistic assumption is false. To derive the reductio, one would require another assumption. One possibility would be a principle of closure under the limit of successive one-step variations, according to which the limit of a sequence of possible worlds obtained by successively removing one obtaining atomic fact at a time is itself a possible world. Another possibility would be an appropriate compactness principle, according to which, if every finite collection of obtaining atomic facts instantiating an object can simultaneously fail to obtain in some possible world, then there is also a possible world in which all such atomic facts simultaneously fail to obtain. One-step independence by itself establishes neither principle.

The reductio established in Section 3 is conditional upon the assumption that, in any possible world, each object is instantiated by only finitely many obtaining atomic facts. Under that assumption, one-step independence is incompatible with necessary instantiation. Whether the *Tractatus* ultimately supports that assumption depends on how the foregoing considerations are weighed.

Objection 3: Proposition 2.0131 concerns only certain kinds of objects

One might argue that proposition 2.0131 should not be read as expressing a general ontological

⁸ For discussions of Wittgenstein's finitistic outlook in the philosophy of mathematics, see, e.g., Mathieu Marion, *Wittgenstein, Finitism, and the Foundations of Mathematics* (Clarendon Press, 1998); Pasquale Frascaola, *Wittgenstein's Philosophy of Mathematics* (Routledge, 1994); and Juliet Floyd, *Wittgenstein's Philosophy of Mathematics* (Cambridge University Press, 2021).

principle concerning all objects. Rather, Wittgenstein merely illustrates, by means of a series of examples, that certain objects necessarily possess some value of a relevant determinable: a spatial object must occupy some location in space, a speck in the visual field must have some colour, a note some pitch, and an object of the sense of touch some degree of hardness. On this interpretation, the proposition establishes only the weaker claim that objects of these kinds must possess some value of the relevant determinable, not that every object must occur in at least one obtaining atomic fact.

Response

Throughout the *Tractatus*, Wittgenstein develops a highly abstract account of objects, states of affairs, and facts. His concern is not with the empirical, psychological, or epistemic question of what kinds of objects there are, but with the logical conditions underlying meaningful representation. It is therefore unlikely that proposition 2.0131 is intended to establish substantive claims about particular kinds of objects. Even less plausible is the suggestion that it advances such claims only with respect to a disparate collection of examples, such as spatial objects, specks in the visual field, notes, and objects of the sense of touch. Rather, these examples are best understood as illustrations of a more general ontological principle that objects always occur in obtaining states of affairs. Read in this way, proposition 2.0131 supports the attribution of the thesis of necessary instantiation to the *Tractatus*.

6. Closing Remarks

In this paper I have distinguished three interpretations of the logical independence of atomic facts in Wittgenstein's *Tractatus*: pairwise independence, one-step independence, and unrestricted combinatorial independence. I have argued that one-step and unrestricted combinatorial independence are incompatible with the doctrine of necessary instantiation and that the *Tractatus* is committed to this doctrine on textual, ontological, and semantic grounds. If these arguments are sound, pairwise independence is the most plausible interpretation of logical independence in the *Tractatus*.

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